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A METHOD FOR ESTIMATING SOME LONGITUDINAL AND LATERAL RIGID-BODY RESPONSES OF AIRPLANES TO CONTINUOUS ATMOSPHERIC TURBULENCE

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16. Abstract

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A METHOD FOR ESTIMATING SOME LONGITUDINAL AND LATERAL RIGID-BODY RESPONSES OF AIRPLANES TO CONTINUOUS ATMOSPHERIC TURBULENCE

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SUMMARY

A method is presented for estimating some lateral and longitudinal rigid-body responses of airplanes to random atmospheric turbulence. Estimates of center-of-gravity load factors and motion responses can be made simply and easily through the use of parametric charts. Sample calculations are given to illustrate the method.

INTRODUCTION

The degree of analytical refinement required in predicting responses (motions, loads, stresses, etc.) of airplanes to atmospheric turbulence depends to a large extent on their size and flexibility. A detailed analysis of the responses of large transport-type airplanes may employ many degrees of freedom for various flight conditions. In order to obtain timely results, the extensive numerical computations prescribed necessitate the use of large-capacity high-speed computer facilities. Otherwise, the response of the comparatively small and rigid executive transports or even first estimates of the response of large transports may be sufficiently described without including the many degrees of freedom associated with dynamic elastic deformation. With this in mind, an investigation was undertaken to develop a simplified procedure for estimating turbulence responses, aimed primarily at small rigid airplanes, that is, airplanes with lowest elastic frequencies three to five times higher than the rigid-body short-period or Dutch roll frequencies.

This study is an extension of the methods reported in references 1 to 4. Reference 1 utilizes a chart with which the root-mean-square value of the normal acceleration of a rigid airplane responding only in plunge (motion in a direction normal to a longitudinal reference axis of the airplane) to the vertical component of turbulence can easily and quickly be obtained. In reference 2, the longitudinal response in pitch and plunge of a small rigid airplane is investigated and parametric charts are presented for making estimates of the effects of the stability characteristics on airplane response statistics.

Reference 3 summarizes early work for both the description of the atmospheric input as well as methods for predicting the airplane response to this input.

The present paper extends reference 2 to cover both uncoupled lateral and longitudinal responses. Some of the results of the analysis reported herein have been previously published in reference 4. The present paper, however, presents a more detailed treatment of the response integrals and the choice of the unsteady lift function is employed. In the analysis, the lateral and longitudinal responses due to independently applied lateral and vertical gust forces are described in terms of two-degree-of-freedom systems. A number of the responses can be expressed in terms of certain definite integrals, herein referred to as response integrals, that appear in both the lateral and longitudinal formulas. As a result of this generality, numerical values of the integrals can be presented in chart form, thereby reducing future analyses to reading values of integrals from charts and entering these values into an appropriate response formula. The charts are similar to those of reference 2 but have been reformulated, mainly by rearrangement of terms in the response equations, to reduce the number of independent parameters upon which they are based.

The basic concepts of the application of random process theory to response calculations are reviewed. The theory is applied, in outline form, to the derivation of a normal-load-factor response formula. The derivation serves to define the generalized response integrals. Additional formulas based on these integrals are given for pitch rate, pitch accelerations, the lateral-load factor, yaw angle, and yaw rate. Parametric charts of the associated integrals are discussed and sample calculations are presented to illustrate chart usage. Accuracy of response calculations to variations of the stability derivatives is investigated. Detailed derivatives of the various response formulas are given in appendix A. An alternate form for the approximation to the classical unsteady lift function used in preparing the charts is discussed in appendix B.

SYMBOLS

$\bar{A}$	airplane-response parameter relating root-mean-square input and output values (subscript refers to response quantity)
AR	aspect ratio
$\tilde{a}$	unsteady lift-force attenuation factor (see table IV)
b	wing span

C_L lift coefficient, $\frac{\text{Lift}}{\frac{1}{2}\rho U^2 S}$

C_m pitching-moment coefficient, $\frac{\text{Pitching moment}}{\frac{1}{2}\rho U^2 S\bar{c}}$

C_n yawing-moment coefficient, $\frac{\text{Yawing-moment}}{\frac{1}{2}\rho U^2 Sb}$

C_Y side-force coefficient, $\frac{\text{Side force}}{\frac{1}{2}\rho U^2 S}$

$\bar{c}$ wing mean aerodynamic chord

e exponential function, 2.718 . . .

g acceleration due to gravity

$H(\omega)$ frequency-response function (subscript refers to response quantity and superscript refers to input force)

I_{yy} mass moment of inertia about lateral axis

I_{zz} mass moment of inertia about vertical axis

$i = \sqrt{-1}$

k reduced frequency, $\frac{\bar{c}\omega}{2U}$

k_0 undamped natural reduced frequency, $\frac{\bar{c}\omega_0}{2U}$

L scale of turbulence

M pitching moment

m airplane mass

N yawing moment

$N(\ ), N_0(\ )$	average number of peaks per unit time which exceed a given level of response and average number of zero crossings per unit time with positive slope (symbol in parenthesis refers to response quantity)
Δn	center-of-gravity normal-load factor
q	pitch velocity, $\dot{\theta}$
$R(\tau)$	autocorrelation function of quantity designated by subscript
R_j	response integrals defined in test (j = 0,2,4,6)
r	yaw velocity, $\dot{\psi}$
r_x	radius of gyration about pitch axis
r_y	radius of gyration about yaw axis
S	wing area
s	relative gust scale, $\frac{2L}{\bar{c}}$
T	general time limit
t	time
U	airplane speed
v	perturbation velocity along y-axis
v_g	lateral component of turbulence velocities
w	perturbation velocity of airplane along z-axis
w_g	vertical component of true turbulence velocities
X	Fourier transform of a quantity designated by subscript
x	longitudinal axis (fig. 7)

Y	side force
y	lateral axis (fig. 7)
$\ddot{y}$	absolute lateral acceleration
Z	vertical-force stability derivative
z	vertical stability axis (fig. 7)
$\ddot{z}$	absolute vertical acceleration
α	angle of attack, $\approx \frac{w}{U}$
β	angle of sideslip, $\approx \frac{v}{U}$
$\bar{\beta}$	frequency ratio, $\frac{\omega}{\omega_0}$
γ	dimensionless damping parameter
γ_α	dimensionless damping parameter for short period
γ_β	dimensionless damping parameter for Dutch roll
ζ	damping ratio
ζ_α	damping ratio for short period
ζ_β	damping ratio for Dutch roll
η_d	multiplier (eq. (4))
κ	mass parameter (table II)
θ	angle of pitch
Λ	sweep back angle of quarter-chord line of lifting surface, degrees
ρ	mass density of air

σ	root-mean-square value of function designated by subscript
τ	time displacement, argument of autocorrelation function
$\Phi(\omega)$	power spectral density of function designated by subscript
$\phi(k), \phi(\omega)$	unsteady lift function
Ψ	unsteady side-force function
ψ	angle of yaw
ψ_s	angle of yaw at zero frequency
ω	circular frequency
ω_0	undamped natural circular frequency

Mathematical conventions:

$\cdot$	first derivative with respect to time
$\cdot\cdot$	second derivative with respect to time
$\lvert\ \rvert$	absolute value of quantity
$[\ ]$	rectangular matrix
$\{\ \}$	column matrix

Dimensionless stability derivatives of airplane are indicated by subscript notation; for example

$$C_{n_r} = \frac{\partial C_n}{\partial\left(\frac{rb}{2U}\right)} \qquad C_{L_\alpha} = \frac{\partial C_L}{\partial\left(\frac{w}{U}\right)}$$

Stability derivatives of airplane are indicated by subscript notation; for example

$$N_r = \frac{\partial N}{\partial\left(\frac{rb}{2U}\right)} \qquad Z_w = \frac{\partial Z}{\partial\left(\frac{w}{U}\right)}$$

STATISTICAL DESCRIPTION OF AIRPLANE RESPONSE

A discussion of the evolution of gust design procedures, description of the turbulence, and response calculation methods as applied to airplanes can be traced by means of references 3 to 7. Since this paper involves an application of the concept of continuous random turbulence, the earlier concepts of the discrete gust are not discussed. In order to provide a foundation for the response calculations, the basic relationships of random process theory as used in the study of continuous turbulence are given a brief definitive treatment herein.

An airplane, upon encountering turbulence, will experience disturbances from its flight path. These disturbances give rise to changes in the center-of-gravity normal-load factor Δn. Since turbulence is considered to be a stationary Gaussian function of time, the airplane response (assuming a linear system) will have the same characteristics. Consequently, both the input gust and the response can be described only in a statistical sense.

Basic Relations

Current methods for determining a design level of the response involve either the concept of the expected exceedances of various response levels or the concept of a most probable maximum level of response. The latter is analogous to the discrete gust approach.

The normal-load-factor response to a vertical-gust input is used herein to illustrate the basic relations involved. Nevertheless, the method is applicable to other responses as well.

Expected exceedances.- An expression for the expected number of times per unit time that the normal-load factor will exceed a given value Δn is given as (see ref. 5)

$$N(\Delta n) = N_0(\Delta n)e^{-\Delta n^2/2\sigma_{\Delta n}^2} \tag{1}$$

where $N_0(\Delta n)$ is the expected number of positive-slope zero crossings per unit time and $\sigma_{\Delta n}$ is the root-mean-square value of the gust-induced normal-load-factor increment.

The parameter $N_0(\Delta n)$ is given by (ref. 8 where N_0 is called $\nu_0{}^+$)

$$N_0(\Delta n) = \frac{1}{2\pi}\frac{\sigma_{\dot{\Delta n}}}{\sigma_{\Delta n}} \tag{2}$$

where $\sigma_{\dot{\Delta n}}$ is the root-mean-square value of the time rate of change of the normal-load factor. The response of the airplane is characterized (ref. 5) by a factor $\bar{A}_{\Delta n}$ which is the ratio of the root-mean-square value of the response to the root-mean-square value of the gust input (in this case the vertical-gust velocity w_g)

$$\bar{A}_{\Delta n} = \frac{\sigma_{\Delta n}}{\sigma_{w_g}} \tag{3}$$

Maximum-response level.- In another method, a quantity $\sigma_{w_g}\eta_d$ is specified in which σ_{w_g} is a root-mean-square gust intensity and η_d is some predetermined multiplier. (See ref. 3.) The design level of the response is thus found from the formula

$$\Delta\eta_{design} = \bar{A}_{\Delta n}\eta_d\sigma_{w_g} \tag{4}$$

It follows then that statistical analysis of the airplane response to continuous turbulence is based on a knowledge of $\bar{A}$ or both $\bar{A}$ and N_0 for all possible flight conditions and airplane configurations.

In the following sections, the relationships of the root-mean-square quantities $\sigma_{\Delta n}$ and $\sigma_{\dot{\Delta n}}$ to the output power spectrum and, in turn, the output spectrum to the response time history is discussed and expressions are given for $\bar{A}$ and N_0.

Root Mean Square and Power Spectra

The root-mean-square value of the normal-load-factor increment $\sigma_{\Delta n}$ (appearing in (eqs. (1) to (3)) is given in terms of a power-spectral-density function $\Phi_{\Delta n}(\omega)$ as (ref. 5)

$$\sigma_{\Delta n} = \left[\int_0^\infty \Phi_{\Delta n}(\omega)\,d\omega\right]^{1/2} \tag{5}$$

Similarly, the root-mean-square value of the time rate of change of Δn is (see ref. 8)

$$\sigma_{\dot{\Delta n}} = \left[\int_0^\infty \omega^2\Phi_{\Delta n}(\omega)\,d\omega\right]^{1/2} \tag{6}$$

The power-spectral-density function $\Phi_{\Delta n}(\omega)$ is in turn related to the random function $\Delta n(t)$ through the Fourier transform of the autocorrelation function (ref. 3)

$$\Phi_{\Delta n}(\omega) = \int_{-\infty}^{\infty} R_{\Delta n}(\tau)e^{-i\omega\tau}\,d\tau \tag{7}$$

where the autocorrelation function is

$$R_{\Delta n}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\Delta n(t)\,\Delta n(t+\tau)\,dt \tag{8}$$

An alternate power-spectral-density function is obtained from the Fourier transform of the response time history

$$\Phi_{\Delta n}(\omega) = \lim_{T\to\infty} \frac{1}{2\pi T} X_{\Delta n}(\omega)\, X_{\Delta n}(-\omega) \tag{9}$$

where

$$X_{\Delta n}(\omega) = \int_{-\infty}^{\infty} \Delta n(t) e^{-i\omega t}\, dt \tag{10}$$

and $X_{\Delta n}(-\omega)$ is the complex conjugate.

Either equation (7) or (9) may be employed to obtain $\Phi_{\Delta n}(\omega)$ when time histories of the normal-load factor are available, such as from flight test measurements. In theoretical studies, however, the output power-spectral-density function $\Phi_{\Delta n}(\omega)$ is obtained in a manner which is described in the next section.

Input-Output Relation

Assume a one-dimensional space-fixed turbulence velocity field, and assume that, for the vertical component, the velocity varies randomly in the direction of flight and is invariant in the spanwise direction. Assume further that the airplane is small enough to be a point transversing the field. The response Δn is then related to the input gust velocity w_g through the input-output relation (ref. 3)

$$\Phi_{\Delta n}(\omega) = \Phi_{w_g}(\omega) \left| H_{\Delta n}^{w_g}(\omega) \right|^2 \tag{11}$$

where $\Phi_{w_g}(\omega)$ is the power spectrum of the vertical component of the turbulence and $H_{\Delta n}^{w_g}(\omega)$ is the frequency-response function of a particular airplane, that is, the steady-state response to a unit sinusoidal input of frequency ω.

Input spectrum.- The spectrum for isotropic homogeneous turbulence has been derived on a theoretical basis by Von Kármán (ref. 9) for the wavelength range of interest in airplane response calculations. Verification of the validity of this spectrum shape has been obtained from experimental evidence for various meteorological conditions. At present, measurements indicate that the spectrum is well represented by the equation suggested in reference 3 where the vertical component is given by

$$\Phi_{w_g}\left(\frac{L\omega}{U}\right) = \frac{L\sigma_{w_g}^{\;2}}{\pi U} \; \frac{1 + \frac{8}{3}\left(1.339\frac{L\omega}{U}\right)^2}{\left[1 + \left(1.339\frac{L\omega}{U}\right)^2\right]^{11/6}} \tag{12}$$

and where L is the integral gust scale defined to be the integral of the autocorrelation function

$$L \equiv \frac{2}{{\sigma_{w_g}}^2} \int_0^\infty R(\tau)\, d\tau \tag{13}$$

Frequency-response function.- The particular form that the frequency-response function takes depends on which response quantity is desired and on the mathematical representation of the airplane. The motion of the airplane consists of rigid-body plunge and pitch in the longitudinal mode and of sideslip and yaw in the lateral mode. The usual two-degree-of-freedom control-fixed rigid-body stability-analysis equations are used for the short-period motion with gust-induced forces and moments added. However, the equations for the lateral mode differ from the usual concepts employed in stability analyses in that roll motion is constrained. Further discussion of this is given in appendix A.

The aerodynamic forces arising from the airplane motion are treated quasistatically with no consideration given to unsteady flow effects other than the use of $C_{m_{\dot{\alpha}}}$. The gust forces are treated quasistatically in the sense that lag in gust and the associated downwash from the wing to tail is not accounted for. However, the attenuation of the gust-force amplitude with increasing frequency is approximated by an unsteady lift function. An unsteady lift function designed to be applicable to finite span wings in compressible flow is discussed in appendix B. The function employed is

$$\left|\phi(k)\right|^2 = e^{-\tilde{a}k} \tag{14}$$

where $\tilde{a}$ characterizes the given planform and flow conditions. Static elastic deformation can be included in determining the aerodynamic forces through modification of the stability derivatives.

Frequency-response functions for the normal-load factor, pitch acceleration, lateral-load factor, and yaw angle have been derived in detail in appendix A. The derivation of the formulas for $\bar{A}_{\Delta n}$ and $N_0(\Delta n)$ is outlined here to develop an expression which defines the response integrals.

Response Parameters $\bar{A}_{\Delta n}$ and $N_0(\Delta n)$

From appendix A, the normal-load-factor frequency response is written in terms of the frequency ratio $\bar{\beta} = \frac{\omega}{\omega_0}$

$$H_{\Delta n}^{w_g}(\bar{\beta}) = \frac{2\zeta_\alpha \omega_{0_\alpha}}{g\gamma_\alpha}\left[\frac{\bar{\beta}^2 - i2\zeta_\alpha\left(1 - \frac{1}{\gamma_\alpha}\right)\bar{\beta}}{1 - \bar{\beta}^2 + i2\zeta_\alpha\bar{\beta}}\right]\phi\left(\omega_{0_\alpha}\ \bar{\beta}\right) \tag{15}$$

where ζ_α is the short-period damping ratio, γ_α is a dimensionless damping force, and $\phi\left(\omega_{0_\alpha}\bar{\beta}\right)$ is an unsteady lift function.

The input power spectrum, equation (12), can be rewritten in terms of a relative-gust-scale parameter $s = \frac{2L}{\bar{c}}$ and the reduced frequency $k_{0_\alpha} = \frac{\bar{c}\omega_{0_\alpha}}{2U}$ as

$$\Phi_{w_g}\left(sk_{0_\alpha}\bar{\beta}\right) = \frac{sk_{0_\alpha}\sigma_{w_g}^2}{\pi\omega_{0_\alpha}} \frac{\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} \tag{16}$$

Upon substitution of equations (15) and (16) into equation (11), there results a formula for the output power spectrum

$$\frac{\Phi_{\Delta n}(\bar{\beta})}{\sigma_{w_g}^2} = \frac{4\zeta_\alpha^2\omega_{0_\alpha}}{g^2\gamma_\alpha^2} \frac{sk_{0_\alpha}}{\pi} \frac{e^{-\tilde{a}k_0\bar{\beta}}\bar{\beta}^2\left[\bar{\beta}^2 + 4\zeta_\alpha^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2\right]\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} \tag{17}$$

Equation (17) is then substituted in equations (5) and (6) and then into equations (2) and (3) to obtain $\bar{A}_{\Delta n}$ and $N_0(\Delta n)$ in the following form

$$\bar{A}_{\Delta n} = \frac{2\zeta_\alpha\omega_{0_\alpha}}{g\gamma_\alpha}\left\{\frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{\bar{\beta}^4 e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}}\,d\bar{\beta}\right.$$
$$\left. + 4\zeta_\alpha^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 \frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{\bar{\beta}^2 e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}}\,d\bar{\beta}\right\}^{1/2} \tag{18}$$

and

$$N_0(\Delta n) = \frac{\zeta_\alpha\omega_{0_\alpha}^2}{g\gamma_\alpha\pi\bar{A}_{\Delta n}\sigma_{w_g}}\left\{\frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{\bar{\beta}^6 e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}}\,d\bar{\beta}\right.$$
$$\left. + 4\zeta_\alpha^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 \frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{\bar{\beta}^4 e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}}\,d\bar{\beta}\right\}^{1/2} \tag{19}$$

Note that $\bar{A}_{\Delta n}$ and $N_0(\Delta n)$ are functions of integrals in the form of $\int \bar{\beta}^j f\left(\bar{\beta},\tilde{a}k_{0_\alpha},sk_{0_\alpha},\zeta\right) d\bar{\beta} = R_j$ and are designated as "response integrals," one for each value of j. In terms of the response integrals, equations (18) and (19) may be rewritten in shortened form as

$$\bar{A}_{\Delta n} = \frac{2\zeta_\alpha \omega_{0_\alpha}}{g\gamma_\alpha}\sqrt{R_4 + 4\zeta_\alpha{}^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_2} \tag{20}$$

and

$$N_0(\Delta n) = \frac{\omega_{0_\alpha}}{2\pi}\sqrt{\frac{R_6 + 4\zeta_\alpha{}^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_4}{R_4 + 4\zeta_\alpha{}^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_2}} \tag{21}$$

where R_j are response integrals, defined as

$$R_j = \frac{sk_0}{\pi}\int_0^\infty \frac{\bar{\beta}^j e^{-\tilde{a}k_0\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_0\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_0\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta} \qquad (j = 0,2,4,6) \tag{22}$$

As the integrals contain the dimensionless arguments ζ, sk_0, and $\tilde{a}k_0$, it is practical to prepare charts of these integrals covering a suitable range of the parameters which will facilitate the calculation of $\bar{A}$ and N_0. An advantage gained by this form of presentation is that four composite charts suffice for the entire calculation for most applications. (The charts are discussed in detail in the next section.) Formulas for $\bar{A}$ and N_0 for other response quantities are given in table I. All of the formulas, both lateral and longitudinal, contain the same response integrals.

PARAMETRIC CHARTS

Evaluation of the response integrals constitute the major numerical effort in gust-response calculations. However, as stated previously, the calculations can be shortened through the use of prepared charts of the integrals as given herein. This section contains a description of the evaluation procedure and scope, discussion of the charts, use of the charts, and example applications.

Evaluation Procedure and Scope

A set of charts has been constructed from a tabulation of the integrals evaluated for the following ranges of the arguments

$$0.004 \leqq \tilde{a}k_0 \leqq 0.4$$
$$1 \leqq sk_0 \leqq 1000$$
$$0.005 \leqq \zeta \leqq 0.5$$

The ranges of parameters are chosen to cover airplanes in the small rigid category.

Numerical integration was employed throughout. The upper limit of integration which is given as infinity in equation (22) is actually a finite frequency ratio $\bar{\beta}$ chosen large enough so that the integral R_4 effectively converged. A check on convergence was made by arbitrarily continuing the integration to a very large upper limit and comparing the results. The integrals R_0, R_2, and R_4 converge rapidly so that the very high frequency components contribute little. However, R_6, which enters only into the calculation for $N_0(\Delta n)$, converges slowly and therefore can become unrealistically unbounded. The numerical integration for R_6 was arbitrarily stopped at the frequency ratio where R_4 $\left(\text{important in } \bar{A}_{\Delta n}\right)$ converged. This procedure follows the suggestion given in reference 10.

Discussion of Charts

The results are presented in figures 1 to 4, where the values of the response integrals are shown plotted against sk_0 and where $\tilde{a}k_0$ and ζ are parameters. The behavior of the integrals R_2 and R_4 is such that a two-order-of-magnitude range, reasonably dense set of values of ζ and $\tilde{a}k_0$ can be conveniently presented without overlapping curves. This presentation permits visual interpolation instead of graphical cross plotting for intermediate values. The same clarity is achieved for R_6 (fig. 4) by omitting some values of ζ.

Use of Charts

In this section, the steps for calculating a numerical value for $\bar{A}$ and N_0 for the normal-load-factor response to vertical gust and the yaw-angle response to lateral gusts will be described. The geometric and inertial properties of the airplane, its nondimensional stability derivatives, and the flight conditions must be available. With these data, the longitudinal and lateral stability characteristics, that is, k_0, ζ, γ, and mass parameter, κ, are computed by utilizing the formulas given in table II.

These parameters, together with the relative-gust-scale parameter $s = \frac{2L}{\bar{c}}$ (mentioned previously) and the unsteady lift-force attenuation factor $\tilde{a}$, described in appendix B, are used to obtain numerical values for the response integrals R_j from figures 1 to 4. The gust scale s will depend on the value of L employed. An L of 762 meters (2500 ft) has been recommended for altitudes above 762 meters. Reasonably accurate values of the response integrals may be taken (by interpolation) from the charts for values of sk_0, ζ, and $\tilde{a}k_0$ not given. The interpolation, which must be logarithmic, is accomplished by utilizing a transparent log scale (a strip cut from log graph paper). It is not deemed necessary that auxiliary cross plots be constructed. Experience has shown that it is best to interpolate for a specific value of $\tilde{a}k_0$, then interpolate for the value of ζ.

Application of the foregoing procedure is illustated by the following sample calculations based on the airplane characteristics listed in table III. The intermediate steps involved in computing the force coefficients will be omitted.

Normal-load factor.- From the data in table III and the equations in table II, the longitudinal stability characteristics are calculated to be $k_{0_\alpha} = 0.035$, $\zeta_\alpha = 0.29$, $\gamma_\alpha = 2.25$, and $sk_{0_\alpha} = 20.8$. Values for the response integrals R_2 and R_4 are taken from figures 2 and 3 by interpolating between $\zeta_\alpha = 0.20$ and 0.50 and are $R_2 = 0.155$ and $R_4 = 0.185$, respectively. The value of $\bar{A}_{\Delta n}$, from equation (20), is then found to be 0.024 g/m/sec.

A design-normal-load factor corresponding to some multiple η_d of a specified root-mean-square gust velocity σ_{w_g} would then be the product of $\bar{A}_{\Delta n}$ and $\eta_d\sigma_{w_g}$ (eq. (4)). In this case, if $\eta_d\sigma_{w_g} = 30.48$ m/sec (100 ft/sec), the design load factor would be 2.4g.

The number of positive-slope zero crossings is similarly calculated from equation (21) by using a value of $R_6 = 1.0$ from figure 4. The value of $N_0(\Delta n)$ is 2.4 crossings per second.

Yaw-angle response.- One of the problems associated with the response of small rigid airplanes to atmospheric turbulence has been the lateral-directional response to side gusts. In this case, the gust forces may not give rise to an overload directly, but rather may initiate Dutch roll response of such a magnitude as to interfere with the maintenance of a proper heading. Discussions of this type of problem may be found in references 11 and 12.

The response integrals provide a means whereby the lateral-motion response for an airplane with given stability characteristics may be qualitatively evaluated. Results from some numerical computations are given. Figure 5 is a plot of $\bar{A}_\psi/\psi_s$ (where values of $\overline{A}$ were determined by use of the charts) against the damping ratio ζ_β where ψ_s is used as a nondimensionalizing factor. The yaw response at zero frequency ψ_s is given by (see appendix A)

$$\psi_s = \frac{1}{U}\left[4\frac{\zeta_\beta^2}{\gamma_\beta}\left(1 - \frac{1}{\gamma_\beta}\right) - 1\right] \tag{23}$$

Thus the ratio $\overline{A}_\psi/\psi_s$ can be interpreted as a gust-amplification factor. Figure 5 is instructive in that the value of the damping ratio ζ_β and the frequency ω_{0_β} which result in a gust-amplification factor in excess of some arbitrary value, say 1.1, can be determined. The gust-response factor increases rapidly for damping-ratio values less than 0.3 and very rapidly for values less than 0.1, thereby suggesting that such an airplane might be difficult to handle in turbulence. This result is in substantial agreement with the conclusion expressed in reference 11.

The data, shown in figure 5, were obtained by arbitrarily varying ζ_β and ω_{0_β} for given values of C_{Y_β}, U, S, m, b, I_{ZZ}, and $\bar{c}$. Therefore, some combinations of ζ_β and ω_{0_β} may not be physically attainable. Nevertheless, figure 5 is included as an illustration of the applicability of the response integrals.

SENSITIVITY OF $\overline{A}$ TO STABILITY-DERIVATIVE VARIATIONS

The method proposed for estimating the gust-response factor $\overline{A}$ has as its basis, the short-period or Dutch roll stability parameters ζ, ω_0, and γ. These parameters are derived from the static and dynamic stability derivatives. The determination of the stability derivatives, whether theoretical or empirical, contains many simplifying assumptions which may adversely affect their accuracy. A study was made to ascertain the sensitivity of the normal-load-factor response parameter $\overline{A}_{\Delta n}$ as a function of a percentage difference in one of the stability derivatives referenced to the value of $\overline{A}_{\Delta n}$ based on a nominal value of the stability derivative. In figure 6, a percentage difference in $\overline{A}$, that is, $\frac{\Delta\overline{A}_{\Delta n}}{\overline{A}_{\Delta n}} \times 100$, is shown as the ordinate plotted against a percentage difference in one of the stability derivatives; for example $\frac{\Delta C_{L_\alpha}}{C_{L_\alpha}} \times 100$ along the abscissa.

Because of the complexity of the function $\overline{A}_{\Delta n}$, the curves were constructed from a numerical rather than an analytical evaluation of these error expressions.

The percentage difference in $\overline{A}_{\Delta n}$ is shown in figure 6 to be almost directly related to a percentage difference in C_{L_α}. Thus, accurate gust-response calculations require an accurate value of C_{L_α}. Otherwise, the influence of C_{m_α} and the dynamic derivatives C_{m_q} and $C_{m_{\dot{\alpha}}}$ is markedly less. Of these derivatives, C_{m_q} has the largest effect. However, it is seen in figure 6 that an error of ±100 percent in the derivative C_{m_q} would result in less than a 20-percent error in $\bar{A}_{\Delta n}$.

Two additional numerical studies were conducted: (1) calculations similar to the preceding for lateral-response factors and (2) the effect of the nonlinear relationship of $\bar{A}_{\Delta n}$ with the stability derivatives. The latter study was made by choosing various values for the nominal values of $C_{L_{\alpha}}$, $C_{m_{\alpha}}$, and so forth. These values were representative of a wide range of airplane sizes. In both studies, the results are essentially the same as those shown in figure 6.

CONCLUDING REMARKS

A simplified procedure for estimating two statistical parameters, $\bar{A}$, the ratio of the root-mean-square response to the root-mean-square input, and N_0, the expected number of positive-slope zero crossings per unit time, is derived herein for the response of a small rigid airplane excited by independently applied vertical and lateral components of random atmospheric turbulence. Formulas are given for the normal-load factor, pitch rate and acceleration, lateral-load factor, and yaw angle and rate. The formulas in each case are based on dynamic systems which can be described by two rigid-body degrees of freedom.

When employing the formulas for making response estimates, the following limitations should be kept in mind:

(1) Tail lag is omitted from the gust forces.

(2) Roll motion is omitted from the lateral response.

(3) Elastic body dynamics are suppressed.

It is reasonable to expect, however, that response estimates thus made will not be in error by more than 10 percent.

The formulas given depend on generalized integral functions referred to as "response integrals". These integrals have as arguments, the damping ratio, a gust-to-airplane scale parameter, an unsteady lift-function attenuation factor, and the undamped reduced frequency. The parameters are associated with either the short-period or Dutch roll mode. The relative gust scale and the unsteady lift-function attenuation factor occur only in combination with the reduced frequency; thereby the number of independent parameters in the integrals number three. The integrals are evaluated over ranges representative of small rigid airplanes and are presented in the form of parametric charts. Because of the nature of the graphical representation, interpolation between values is practical, thus facilitating the response calculations.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., May 11, 1971.

APPENDIX A

DERIVATION OF RESPONSE FORMULAS

The frequency-response functions required by the input-output relations, equation (11), are obtained by solving the equations of motion of the airplane for independently applied vertical gust forces. The gust forces result from flight of the airplane through a one-dimensional gust field (ref. 3). The equations of motion, taken from references 13 and 14, are based on the assumptions commonly made in stability analyses of rigid airplanes. In stability analyses, the airplane's motion is described by six coordinates, three translations, and three rotations referenced to the stability axes shown in figure 7. If a plane of symmetry is assumed to exist, the terms that couple longitudinal and lateral motion can be neglected and for linearized equations, the six-degree-of-freedom system is separated into two groups, the longitudinal and the lateral. Consequently, each system can be analyzed separately as a two- or three-degree-of-freedom system.

Longitudinal Motion

The three degrees of freedom comprising the longitudinal motion are vertical translation (plunge), pitch rotation, and horizontal translation in the direction of flight. Perturbations in horizontal translation have been shown to have little effect on the short-period response; therefore, the longitudinal case will be treated herein as a two-degree-of-freedom system. These equations, as given in reference 14, are

$$\begin{bmatrix} i\omega - Z_w & -i\omega U \\ -i\omega M_{\dot{w}} - M_w & -\omega^2 - i\omega M_q \end{bmatrix} \begin{Bmatrix} w \\ \theta \end{Bmatrix} = \phi(\omega) \begin{Bmatrix} Z_w \\ M_w \end{Bmatrix} w_g \tag{A1}$$

where

$$Z_w = \frac{-\rho US}{2m} C_{L_\alpha}$$

$$M_{\dot{w}} = \frac{\rho S\bar{c}^2}{4I_{yy}} C_{m_{\dot{\alpha}}}$$

$$M_w = \frac{\rho US\bar{c}}{2I_{yy}} C_{m_\alpha}$$

$$M_q = \frac{\rho US\bar{c}^2}{4I_{yy}} C_{m_q}$$

In the gust forces on the right-hand side of equation (A1), the unsteady lift function $\phi(\omega)$ represents the manner in which both the aerodynamic forces and moments are assumed to vary with the frequency of the gust input velocity. Note that the flight path is assumed to be approximately horizontal so that the plunging velocity w and the vertical-gust-velocity component w_g may be taken to be collinear. A discussion of a suitable choice of $\phi(\omega)$ is given in appendix B.

The frequency-response function for w or θ is obtained in the usual manner by solving equation (A1) for a unit gust velocity w_g. Then

$$\begin{Bmatrix} w \\ \theta \end{Bmatrix} = \frac{\begin{bmatrix} -\omega^2 - i\omega M_q & i\omega U \\ i\omega M_{\dot{w}} + M_w & i\omega - Z_w \end{bmatrix} \begin{Bmatrix} Z_w \\ M_w \end{Bmatrix} \phi(\omega)}{i\omega\left[-\omega^2 - i\omega\left(M_q + Z_w + UM_{\dot{w}}\right) + \left(Z_w M_q - UM_w\right)\right]} \tag{A2}$$

The frequency-response functions are then

$$H_w^{w_g}(\omega) = \frac{\left[\left(-\omega^2 - i\omega M_q\right)Z_w + i\omega UM_w\right]\phi(\omega)}{i\omega\left[-\omega^2 - i\omega\left(M_q + Z_w + UM_{\dot{w}}\right) + \left(Z_w M_q - UM_w\right)\right]} \tag{A3}$$

and

$$H_\theta^{w_g}(\omega) = \frac{\left[\left(i\omega M_{\dot{w}} + M_w\right)Z_w + \left(i\omega - Z_w\right)M_w\right]\phi(\omega)}{i\omega\left[-\omega^2 - i\omega\left(M_q + Z_w + UM_{\dot{w}}\right) + \left(Z_w M_q - UM_w\right)\right]} \tag{A4}$$

Equations (A3) and (A4) can be simplified and put into another form if the following notation is employed

$$\bar{\beta} = \frac{\omega}{\omega_{0_\alpha}}$$

$$\omega_{0_\alpha} = \left(Z_w M_q - UM_w\right)^{1/2}$$

$$2\zeta_\alpha \omega_{0_\alpha} = -\left(M_q + Z_w + UM_{\dot{w}}\right)$$

$$\gamma_\alpha = \frac{M_q + Z_w + UM_{\dot{w}}}{Z_w}$$

Equations (A3) and (A4) are now

$$H_w^{w_g}(\bar{\beta}) = \frac{-\left(\frac{2\zeta_\alpha \bar{\beta}}{\gamma_\alpha} i + 1\right)}{1 - \bar{\beta}^2 + i2\zeta_\alpha \bar{\beta}} \phi\left(\omega_{0_\alpha} \bar{\beta}\right) \tag{A5}$$

$$H_\theta^{w_g}(\bar{\beta}) = \frac{1}{U} \frac{\left[4\frac{\zeta_\alpha^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]}{1 - \bar{\beta}^2 + i2\zeta_\alpha \bar{\beta}} \phi\left(\omega_{0_\alpha} \bar{\beta}\right) \tag{A6}$$

A response parameter of major interest is the vertical acceleration $\ddot{z}$. Since the stability axes are rotating, the absolute acceleration is given by $i\omega(w - U\theta)$. The vertical-acceleration frequency response is the sum of the frequency responses $H_w^{w_g}(\bar{\beta})$ and $H_\theta^{w_g}(\bar{\beta})$. The resulting expression divided by g gives the normal-load-factor frequency-response function

$$H_{\Delta n}^{w_g}(\bar{\beta}) = \frac{2\zeta_\alpha \omega_{0_\alpha}}{g\gamma_\alpha} \left[\frac{\bar{\beta}^2 - i2\zeta_\alpha \left(1 - \frac{1}{\gamma_\alpha}\right)\bar{\beta}}{1 - \bar{\beta}^2 + i2\zeta_\alpha \bar{\beta}}\right] \phi\left(\omega_{0_\alpha} \bar{\beta}\right) \tag{A7}$$

Equation (11) requires the square of the absolute value of the frequency-response function

$$\left|H_{\Delta n}^{w_g}(\bar{\beta})\right|^2 = \frac{4\zeta_\alpha^2 \omega_{0_\alpha}^2}{g^2 \gamma_\alpha^2} \left[\frac{\bar{\beta}^4 + 4\zeta_\alpha^2 \left(1 - \frac{1}{\gamma_\alpha}\right)^2 \bar{\beta}^2}{\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha^2 \bar{\beta}^2}\right] \left|\phi\left(\omega_{0_\alpha} \bar{\beta}\right)\right|^2 \tag{A8}$$

The squares of the absolute values of the frequency-response functions of $q = i\omega\theta$ and $\dot{q} = -\omega^2\theta$ are similarly

$$\left|H_q^{w_g}(\bar{\beta})\right|^2 = \frac{\omega_{0_\alpha}^2 \bar{\beta}^2}{U^2} \frac{\left[4\frac{\zeta_\alpha^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right)^2 - 1\right]^2}{\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha^2 \bar{\beta}^2} \left|\phi\left(\omega_{0_\alpha} \bar{\beta}\right)\right|^2 \tag{A9}$$

and

$$\left|H_{\dot{q}}^{w_g}(\bar{\beta})\right|^2 = \frac{\omega_{0_\alpha}{}^4 \bar{\beta}^4}{U^2} \frac{\left[4\frac{\zeta_\alpha{}^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right)^2 - 1\right]^2}{\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha{}^2\bar{\beta}^2} \left|\phi\left(\omega_{0_\alpha}\bar{\beta}\right)\right|^2 \tag{A10}$$

The frequency-response functions (A8), (A9), or (A10), with $\left|\phi\left(\omega_{0_\alpha}\bar{\beta}\right)\right|^2 = e^{-\tilde{a}k_0\bar{\beta}}$ (see appendix B), together with the input spectrum, equation (12), are substituted into equations (11), (5), and (3) to obtain

$$\bar{A}_{\Delta n} = \frac{2\zeta_\alpha\omega_{0_\alpha}}{g\gamma_\alpha}\left\{\frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\bar{\beta}^2\left[\bar{\beta}^2 + 4\zeta_\alpha{}^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2\right]\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha{}^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta}\right\}^{1/2} \tag{A11}$$

$$\bar{A}_q = \frac{\omega_{0_\alpha}\left[4\frac{\zeta_\alpha{}^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]}{U}\left\{\frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\bar{\beta}^2\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha{}^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta}\right\}^{1/2} \tag{A12}$$

$$\bar{A}_{\dot{q}} = \frac{\omega_{0_\alpha}{}^2\left[4\frac{\zeta_\alpha{}^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]}{U}\left\{\frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\bar{\beta}^4\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)\right]^2}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha{}^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta}\right\}^{1/2} \tag{A13}$$

Values of the response parameters $\bar{A}$ could be calculated for given values of the airplane characteristics $\tilde{a}$, k_0, ζ, and γ and the relative-gust-scale parameter s by evaluating the infinite integral. It was found advantageous, however, to reduce the integrands to functions of the three parameters $\tilde{a}k_0$, sk_0, and ζ. Equation (A11) becomes

$$\bar{A}_{\Delta n} = \frac{2\zeta_\alpha \omega_{0_\alpha}}{g\gamma_\alpha}\left\{\frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{\bar{\beta}^4 e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha{}^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta}\right.$$

$$\left. + 4\zeta_\alpha{}^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 \frac{sk_{0_\alpha}}{\pi}\int_0^\infty \frac{\bar{\beta}^2 e^{-\tilde{a}k_{0_\alpha}\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\alpha{}^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_{0_\alpha}\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta}\right\}^{1/2}$$

The integrands are referred to as response integrals and are of the general form

$$R_j = \frac{sk_0}{\pi}\int_0^\infty \frac{\bar{\beta}^j e^{-\tilde{a}k_0\bar{\beta}}\left[1 + \frac{8}{3}\left(1.339sk_0\bar{\beta}\right)^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta^2\bar{\beta}^2\right]\left[1 + \left(1.339sk_0\bar{\beta}\right)^2\right]^{11/6}} d\bar{\beta} \qquad (j = 0,2,4,6)$$

The formulas for the parameters $\bar{A}$ are then given in terms of the response integrals as follows:

$$\bar{A}_{\Delta n} = \frac{2\zeta_\alpha \omega_{0_\alpha}}{g\gamma_\alpha}\left[R_4 + 4\zeta_\alpha{}^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_2\right]^{1/2} \tag{A14}$$

$$\bar{A}_q = \frac{\omega_{0_\alpha}}{U}\left[4\frac{\zeta_\alpha{}^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]\left(R_2\right)^{1/2} \tag{A15}$$

$$\bar{A}_{\dot{q}} = \frac{\omega_{0_\alpha}{}^2}{U}\left[4\frac{\zeta_\alpha{}^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]\left(R_4\right)^{1/2} \tag{A16}$$

Lateral Motion

In order to facilitate estimates of the lateral response, it is desirable that the number of parameters be held to a minimum. Consequently, a study was made to determine under what conditions the three-degree-of-freedom lateral motion of a rigid fixed-control-surface airplane (i.e., yaw, roll, and sideslip) could be represented as a two-degree-of-freedom system (yaw and sideslip). Previous work, notably references 15 to 17, indicate that this simplification is possible. The assumptions that are made in reference 17 are that the airplane roll motion is minimized by the pilot, the vertical-tail loads were due to side gusts only, and the pilot had little effect on loads. It is stated in reference 17 that flight records indicated that very small roll angles resulted from gusts, thus lending support to the first assumption. In reference 13, it is suggested that the side force due to yawing velocity can be neglected for most configurations. Therefore, the following assumptions are made herein:

(1) Airplane does not roll.

(2) Lateral response is due to side gust only.

(3) Side force due to yawing velocity is negligible.

Assumptions (1) and (3) were tested by a comparison of the response calculated for a representative small-transport configuration. These calculations were based on a three-degree-of-freedom model, a two-degree-of-freedom model, and the two-degree-of-freedom model with yaw side force omitted. An indication of the generality of assumptions (1) and (3) was further obtained by varying the ratios of inertial and aerodynamic yaw-roll coupling terms. These studies confirmed the validity of assumptions (1) and (3) for the small-transport design. Assumption (2) is made solely on the basis of the work of reference 16.

On the basis of the three-degree-of-freedom studies discussed previously, lateral-response equations are derived for a two-degree-of-freedom model consisting of yaw and sideslip. In these equations, roll response and the side load due to yawing are neglected. In matrix form, the equations of motion are

$$\begin{bmatrix} i\omega - Y_v & i\omega U \\ -N_v & -\omega^2 - i\omega N_r \end{bmatrix} \begin{Bmatrix} v(\omega) \\ \psi(\omega) \end{Bmatrix} = \Psi(\omega) \begin{Bmatrix} -Y_v \\ -N_v \end{Bmatrix} v_g \tag{A17}$$

where

$$Y_v = \frac{\rho US}{2m} C_{Y_\beta}$$

$$N_v = \frac{\rho USb}{2I_{ZZ}} C_{n_\beta}$$

and

$$N_r = \frac{\rho USb^2}{4I_{ZZ}} C_{n_r}$$

In equation (A17), the function $\Psi(\omega)$ represents the unsteady lift force and moment variation with the frequency of the gust-velocity component $v_g(\omega)$ of the lateral one-dimensional gust field. The gust-velocity component $v_g(\omega)$ is, as in the longitudinal case, assumed to be collinear with sideslip velocity v.

Solutions for the variables $v(\omega)$ and $\psi(\omega)$ are obtained from equation (A17) by assuming a unit gust velocity $v_g(\omega)$. Thus

$$\begin{Bmatrix} v(\omega) \\ \psi(\omega) \end{Bmatrix} = \frac{\begin{bmatrix} -\omega^2 - i\omega N_r & -i\omega U \\ N_v & i\omega - Y_v \end{bmatrix} \begin{Bmatrix} -Y_v \\ -N_v \end{Bmatrix}}{i\omega\left[-\omega^2 - i\omega\left(N_r + Y_v\right) + \left(N_r Y_v + UN_v\right)\right]} \Psi(\omega) v_g \tag{A18}$$

The frequency-response functions are then

$$H_v^{v_g}(\omega) = \frac{\left[\left(-\omega^2 - i\omega N_r\right)\left(-Y_v\right) + i\omega UN_v\right]\Psi(\omega)}{i\omega\left[-\omega^2 - i\omega\left(N_r + Y_v\right) + \left(N_r Y_v + UN_v\right)\right]} \tag{A19}$$

$$H_\psi^{v_g}(\omega) = \frac{\left[-N_v Y_v - \left(i\omega - Y_v\right)N_v\right]\Psi(\omega)}{i\omega\left[-\omega^2 - i\omega\left(N_r + Y_v\right) + \left(N_r Y_v + UN_v\right)\right]} \tag{A20}$$

These equations can be simplified in a manner similar to the treatment of the longitudinal responses. The following notation is employed

$$\bar{\beta} = \frac{\omega}{\omega_{0_\beta}}$$

$$\omega_{0_\beta} = \left(N_r Y_v + UN_v\right)^{1/2}$$

$$2\zeta_\beta \omega_{0_\beta} = -\left(N_r + Y_v\right)$$

$$\gamma_\beta = \frac{N_r + Y_v}{Y_v}$$

Equations (A19) and (A20) become

$$H_v^{v_g}(\bar{\beta}) = \frac{\left(1 + i2\frac{\zeta_\beta}{\gamma_\beta}\bar{\beta}\right)}{1 - \bar{\beta}^2 + i2\zeta_\beta\bar{\beta}}\Psi\left(\omega_{0_\beta}\bar{\beta}\right) \tag{A21}$$

$$H_\psi^{v_g}(\bar{\beta}) = \frac{1}{U}\frac{\left[4\frac{\zeta_\beta{}^2}{\gamma_\beta}\left(1 - \frac{1}{\gamma_\beta}\right) - 1\right]}{1 - \bar{\beta}^2 + i2\zeta_\beta\bar{\beta}}\Psi\left(\omega_{0_\beta}\bar{\beta}\right) \tag{A22}$$

The absolute lateral acceleration $\ddot{y}$ as seen by an accelerometer mounted at the airplane center of gravity is given by

$$\ddot{y} = \dot{v} + U\dot{\psi} = i\omega_{0_\beta}\bar{\beta}\left(v + U\psi\right)$$

Then

$$H_{\ddot{y}/g}^{v_g}(\bar{\beta}) = \frac{2\zeta_\beta\omega_{0_\beta}}{g\gamma_\beta}\frac{\left[-\bar{\beta}^2 + i2\zeta_\beta\left(1 - \frac{1}{\gamma_\beta}\right)\bar{\beta}\right]}{1 - \bar{\beta}^2 + i2\zeta_\beta\bar{\beta}}\Psi\left(\omega_{0_\beta}\bar{\beta}\right) \tag{A23}$$

With $\left|\Psi\left(\omega_{0_\beta}\bar{\beta}\right)\right|^2 \rightarrow e^{-\tilde{a}k_{0_\beta}\bar{\beta}}$ (appendix B), the square of the absolute value of equation (A23) is

$$\left|H_{\ddot{y}/g}^{v_g}(\bar{\beta})\right|^2 = \frac{4\zeta_\beta{}^2\omega_{0_\beta}{}^2}{g^2\gamma_\beta{}^2}\frac{\left[\bar{\beta}^4 + 4\zeta_\beta{}^2\left(1 - \frac{1}{\gamma_\beta}\right)^2\bar{\beta}^2\right]}{\left[\left(1 - \bar{\beta}^2\right)^2 + 4\zeta_\beta{}^2\bar{\beta}^2\right]}e^{-\tilde{a}k_{0_\beta}\bar{\beta}}$$

Because the lateral-response equations have the same form as the longitudinal equations, the formulas for the root-mean square lateral response can be written analogously as

$$\bar{A}_{\ddot{y}/g} \sim \bar{A}_{\Delta n}$$

$$\bar{A}_\psi \sim \bar{A}_\theta$$

$$\bar{A}_{\ddot{\psi}} \sim \bar{A}_{\dot{q}}$$

APPENDIX B

SUGGESTED APPROXIMATION FOR UNSTEADY LIFT FUNCTION

The aerodynamic forces and moments resulting from the motion of the airplane are based on the quasisteady approximation in which, in effect, the forces correspond to steady angles of attack and to constant velocities. This approximation is possible because the short-period and Dutch roll motion is slow in time (low frequency). Otherwise, the incident gust-velocity time function contains high-frequency components (rapidly changing) that are inadequately described by the quasisteady approximation. Another function, generally referred to as the unsteady lift function, is therefore employed to describe the manner, in the frequency domain, in which the gust forces vary with frequency. Different functions have been devised to approximate the unsteady lift. In reference 2, the function suggested in reference 18 for incompressible two-dimensional aerodynamics was used. Thus,

$$|\phi(k)|^2 = \frac{1}{1 + 2\pi k} = 1 - 2\pi k + (2\pi k)^2 - \ldots \tag{B1}$$

In an effort to obtain an improved approximation useful for finite span wings (three-dimensional aerodynamics) in compressible flow, the unsteady lift as a function of reduced frequency k was calculated for various planforms and Mach numbers. The kernel function method described in reference 19 was used. The results of the numerical analysis together with additional data taken from reference 20 are shown in figures 8 and 9. It is apparent from figure 8 that the functions when plotted on semilog paper are nearly straight lines. This suggests that the data might be fitted to an exponential function

$$|\phi(k)|^2 = e^{-\tilde{a}k} = 1 - \tilde{a}k + \frac{\tilde{a}^2 k^2}{2!} - \ldots \tag{B2}$$

The coefficient $\tilde{a}$ is arrived at from the semilog plots of $|\phi(k)|^2$ against k such as figure 8. First, a best fit straight line is drawn through the curve and the value of $|\phi(k)|^2$ for $k = 1$ is noted. Then from a table of exponential functions, a value for $\tilde{a}$ is found which corresponds to $|\phi(1)|^2$. It is recognized that the straight line chosen in this manner is somewhat arbitrary, but it was not considered necessary to go to a curve-fitting routine. The values of $\tilde{a}$ thus derived are given in table IV. A comparison between $|\phi(k)|^2 = e^{-\tilde{a}k}$ and $|\phi(k)|^2 = \frac{1}{1 + 2\pi k}$ as an approximation to the calculated unsteady lift function is shown in figure 8. The calculated results are more closely matched by the exponential than by the function $|\phi(k)|^2 = \frac{1}{1 + 2\pi k}$, particularly in the interval $0 < k < 0.5$.

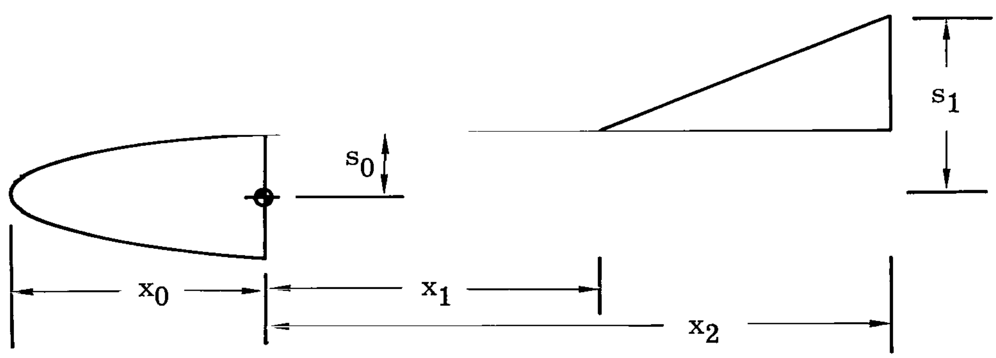
A method of computing the unsteady aerodynamic forces and moments due to sinusoidal lateral gusts impinging on the fuselage-fin combination is presented in reference 16. The method is based on slender-body theory given in reference 21 which presents an expression for the normal force per unit streamwise length of a flat plate of increasing area but high fineness ratio. Equations for the side force and yawing-moment coefficients due to a lateral gust are

$$C_{Y_\beta}(\omega) = \frac{2\pi}{S}\left\{\frac{2s_0^2}{k_0^2}\left[1 - \left(1 - ik_0\right)e^{ik_0}\right] + \left(\frac{s_1 - s_0}{k_2 - k_1}\right)^2\left[e^{-ik_1} - \left(1 - ik_1 + ik_2\right)e^{-ik_2}\right]\right\} \tag{B3}$$

$$C_{n_\beta}(\omega) = \frac{2\pi}{Sb}\left\{-\frac{2x_0 s_0^2}{k_0^3}\left[\left(2k_0 - ik_0^2 + 2i\right)e^{ik_0} - 2i\right] + \frac{\left(x_1 - x_2\right)\left(s_1 - s_0\right)^2}{\left(k_2 - k_1\right)^2}\left[\left(2k_2 - k_1 - 2i - ik_1k_2 + ik_2^2\right)e^{-ik_2} - \left(k_1 - 2i\right)e^{-ik_1}\right]\right\}$$

$$\left(k_n = \frac{\omega x_n}{U};\quad n = 0,1,2\right) \tag{B4}$$

The parameters contained in equations (B3) and (B4) are defined by the following sketch



These equations were evaluated for two airplane configurations. The results are presented in figure 10 as the square of the magnitude of the unsteady normal-force function plotted against the reduced frequency k. The reduced parameter k_n in equation (B3) and (B4) are related to k as

$$k_n = \frac{\bar{c}\omega}{2U}\left(\frac{2x_n}{\bar{c}}\right) = \left(\frac{2x_n}{\bar{c}}\right)k \tag{B5}$$

The results given are consequently a function of the particular configuration. Curves could be presented for a wide range of values of x_n, s_0, and s_1; however, the accuracy of the approach is not considered sufficient to warrant such a refinement.

For the airplane configuration studied, assumed representative of the class considered herein, the unsteady lift functions for both the wing and fuselage-fin combination exhibit similar behavior and are therefore both approximated by the exponential function. Values of $\tilde{a}$ for the vertical case can be chosen by using table IV as a guide; however, for the lateral case an appropriate value would be between 0.8 and 1.2. Since the factor $\tilde{a}$ occurs in the response integrals only in combination with k_0, the product $\tilde{a}k_0$ is treated as a single parameter in the charts (figs. 1 to 4). Response calculations can be made to include corrections for planform and Mach number by utilizing the suitably shaped exponential to represent the unsteady lift function. The attenuation factors $\tilde{a}$ given are based on subsonic flow. If computation of the gust response of an airplane flying supersonically is desired, appropriate values for $\tilde{a}$ would have to be obtained.

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TABLE I.- RESPONSE FORMULAS

Response Quantity	$\bar{A}$	N_0
Normal-load factor	$\bar{A}_{\Delta n} = \frac{2\zeta_\alpha \omega_{0_\alpha}}{g\gamma_\alpha}\sqrt{R_4 + 4\zeta_\alpha^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_2}$	$N_0(\Delta n) = \frac{\omega_{0_\alpha}}{2\pi}\sqrt{\frac{R_6 + 4\zeta_\alpha^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_4}{R_4 + 4\zeta_\alpha^2\left(1 - \frac{1}{\gamma_\alpha}\right)^2 R_2}}$
Pitch-angle rate	$\bar{A}_{\dot{\theta}} = \frac{\omega_{0_\alpha}}{U}\left[4\frac{\zeta_\alpha^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]\sqrt{R_2}$	$N_0(\dot{\theta}) = \frac{\omega_{0_\alpha}}{2\pi}\sqrt{\frac{R_4}{R_2}}$
Pitch-angle acceleration	$\bar{A}_{\ddot{\theta}} = \frac{\omega_{0_\alpha}^2}{U}\left[4\frac{\zeta_\alpha^2}{\gamma_\alpha}\left(1 - \frac{1}{\gamma_\alpha}\right) - 1\right]\sqrt{R_4}$	$N_0(\ddot{\theta}) = \frac{\omega_{0_\alpha}}{2\pi}\sqrt{\frac{R_6}{R_4}}$
Lateral-load factor	$\bar{A}_{\ddot{y}/g} = \frac{2\zeta_\beta \omega_{0_\beta}}{g\gamma_\beta}\sqrt{R_4 + 4\zeta_\beta^2\left(1 - \frac{1}{\gamma_\beta}\right)^2 R_2}$	$N_0(\ddot{y}/g) = \frac{\omega_{0_\beta}}{2\pi}\sqrt{\frac{R_6 + 4\zeta_\beta^2\left(1 - \frac{1}{\gamma_\beta}\right)^2 R_4}{R_4 + 4\zeta_\beta^2\left(1 - \frac{1}{\gamma_\beta}\right)^2 R_2}}$
Yaw angle	$\bar{A}_{\psi} = \frac{1}{U}\left[4\frac{\zeta_\beta^2}{\gamma_\beta}\left(1 - \frac{1}{\gamma_\beta}\right) - 1\right]\sqrt{R_0}$	$N_0(\psi) = \frac{\omega_{0_\beta}}{2\pi}\sqrt{\frac{R_2}{R_0}}$
Yaw-angle rate	$\bar{A}_{\dot{\psi}} = \frac{\omega_{0_\beta}}{U}\left[4\frac{\zeta_\beta^2}{\gamma_\beta}\left(1 - \frac{1}{\gamma_\beta}\right) - 1\right]\sqrt{R_2}$	$N_0(\dot{\psi}) = \frac{\omega_{0_\beta}}{2\pi}\sqrt{\frac{R_4}{R_2}}$

TABLE II.- FORMULAS FOR STABILITY CHARACTERISTICS

	Longitudinal (α)	Lateral (β)
k_0	$\frac{\bar{c}}{r_x}\sqrt{-\frac{1}{\kappa C_{L_\alpha}}\left(-\frac{2}{\kappa}C_{m_q} + C_{m_\alpha}\right)}$	$\frac{\bar{c}}{r_y}\sqrt{\frac{1}{\kappa C_{Y_\beta}}\left(-\frac{2}{\kappa}C_{n_r} + C_{n_\beta}\right)}$
ζ	$\frac{1}{\kappa k_0}\left[1 - \left(\frac{\bar{c}}{r_x}\right)^2 \frac{C_{m_q} + C_{m_\alpha}}{2C_{L_\alpha}}\right]$	$-\frac{1}{\kappa k_0}\left[1 + \left(\frac{b}{r_y}\right)^2 \frac{C_{n_r}}{2C_{Y_\beta}}\right]$
κ	$\frac{8m}{\rho S\bar{c}C_{L_\alpha}}$	$\frac{8m}{\rho SbC_{Y_\beta}}$
γ	$\zeta\kappa k_0$	$\zeta\kappa k_0$

TABLE III.- AIRPLANE CHARACTERISTICS AND FLIGHT CONDITIONS USED IN SAMPLE CALCULATIONS

U, m/sec	244
Altitude, km	4.95
ρ, kg/m^3	0.7415
m, kg	7880
I_{zz}, kg-m^2	69 800
I_{yy}, kg-m^2	24 900
S, m^2	31.8
$\bar{c}$, m	2.6
$\tilde{a}$ –	
Longitudinal	2.0
Lateral	0.8
C_{L_α}	5.17
C_{m_α}	-1.03
$C_{m_{\dot{\alpha}}}$	-3.2
C_{m_q}	-6.8
C_{Y_β}	-0.792
C_{n_β}	0.117
C_{n_r}	-0.140
b, m	13.6
L, m	762

TABLE IV.- ATTENUATION FACTORS $\tilde{a}$

M \ Λ	0°	30°	35°	45°	AR
0.2	-0.96	----	-1.05	----	4.00
↓	-1.28	----	-1.55	----	6.00
↓	-1.40	----	-2.10	----	9.43
.4	-0.96	-1.00	-1.00	----	4.00
↓	-1.28	-1.35	-1.35	----	6.00
↓	-1.44	----	----	----	9.43
.9	-1.40	----	----	----	4.00
↓	-2.06	-2.06	-2.06	----	6.00
↓	-2.90	----	----	-2.1	9.43

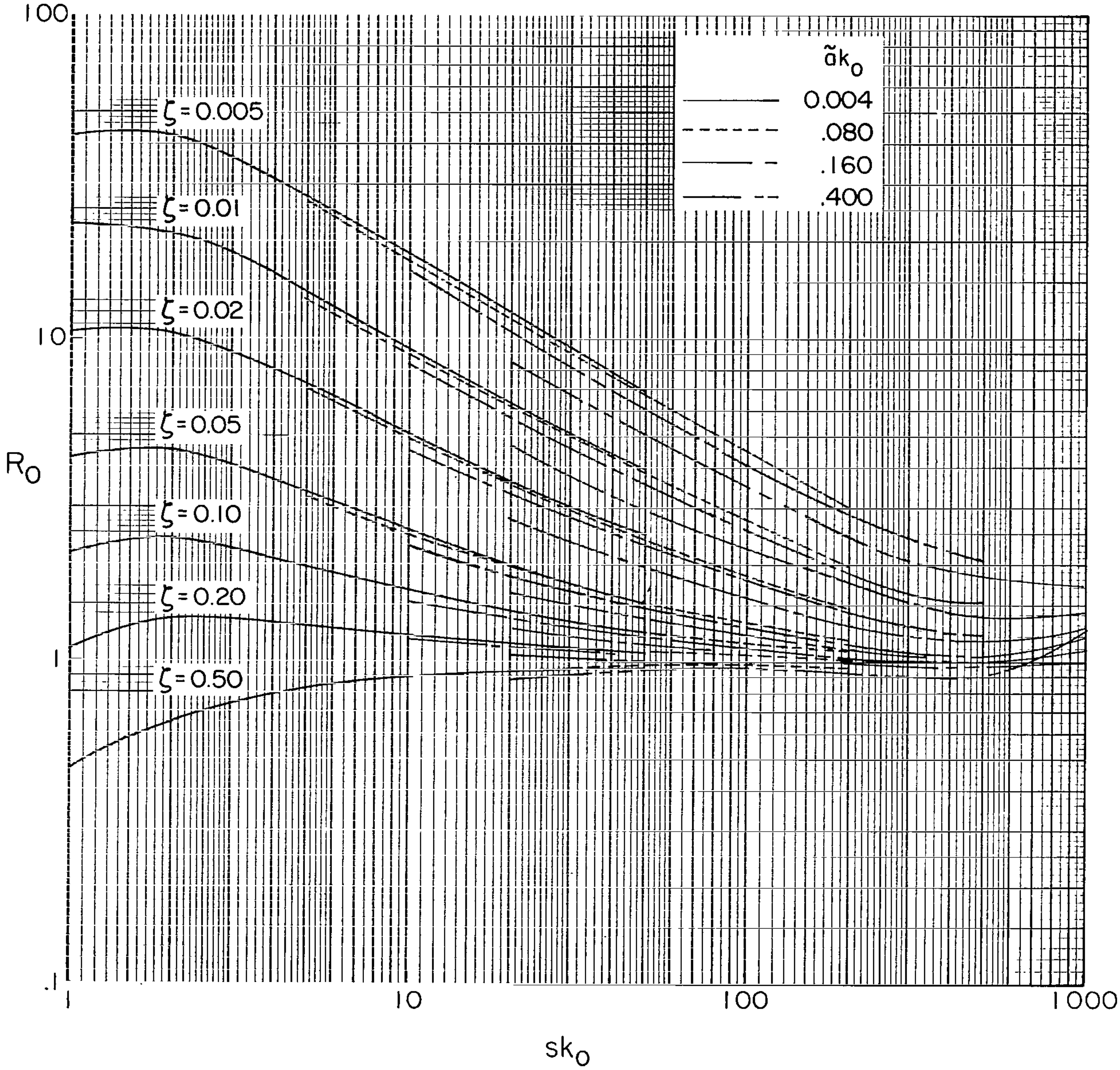


Figure 1.- Response integral R_0 as function of sk_0 for various values of ζ and $\tilde{a}k_0$.

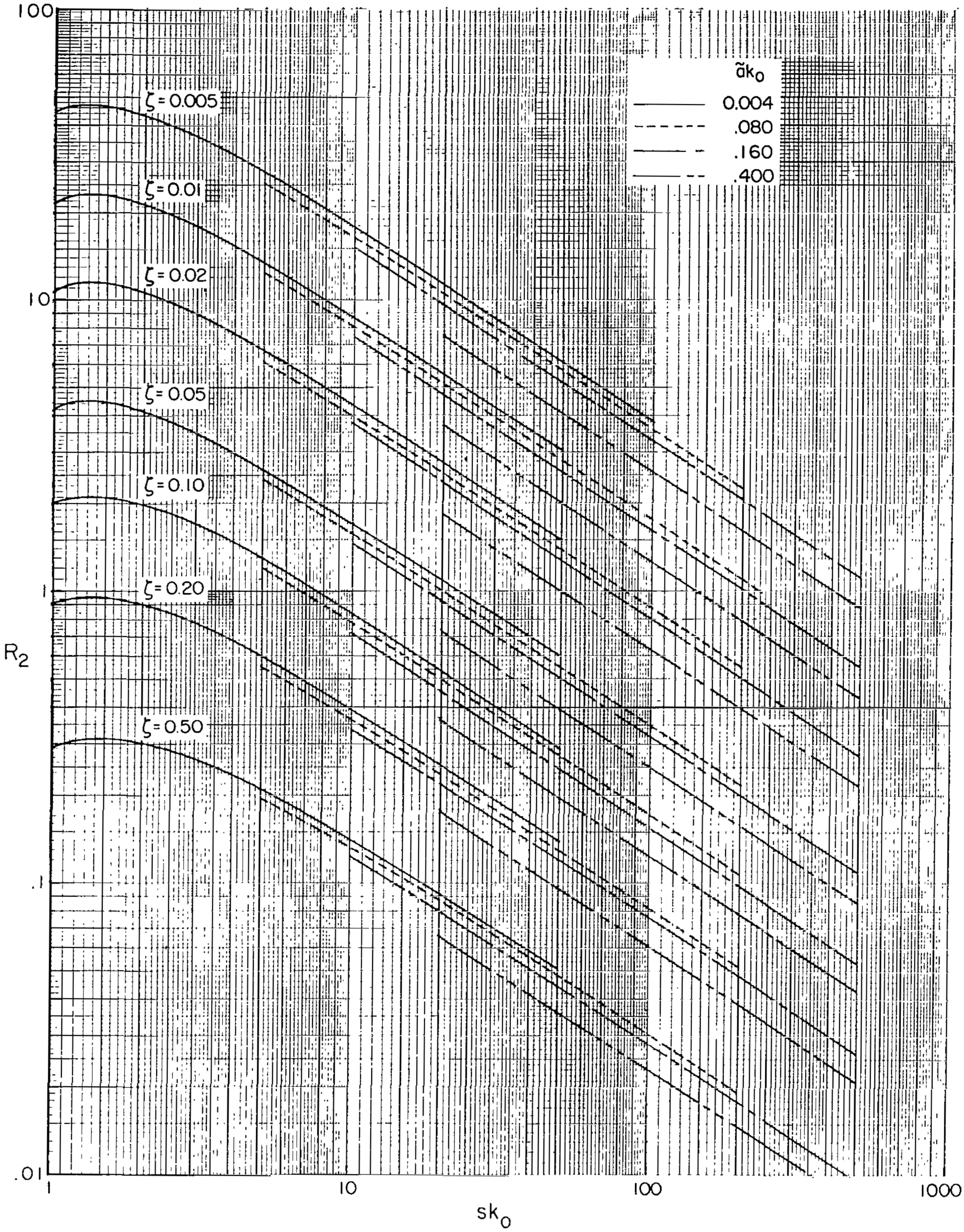


Figure 2.- Response integral R_2 as function of sk_0 for various values of ζ and $\tilde{a}k_0$.

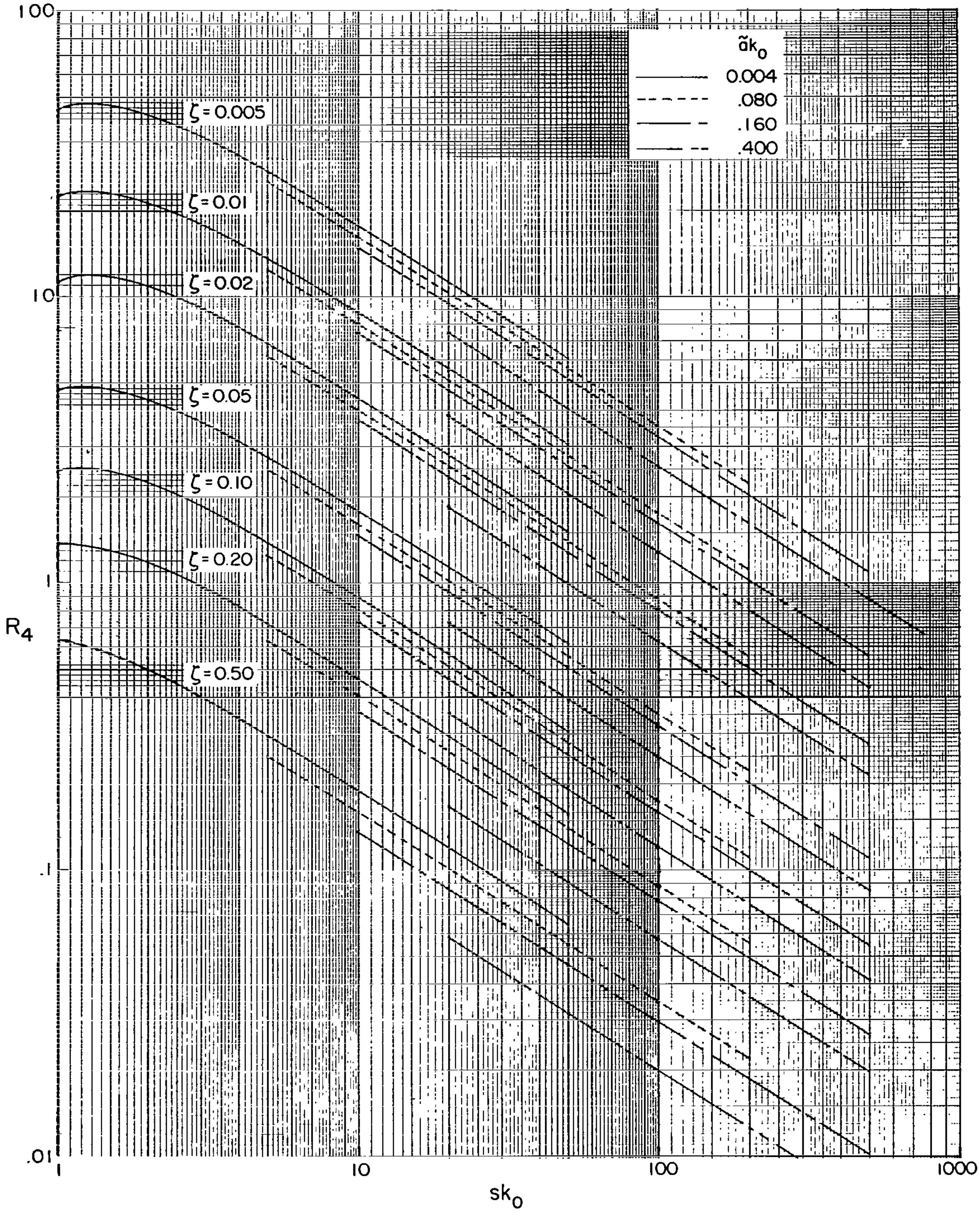


Figure 3.- Response integral R_4 as function of sk_0 for various values of ζ and $\tilde{a}k_0$.

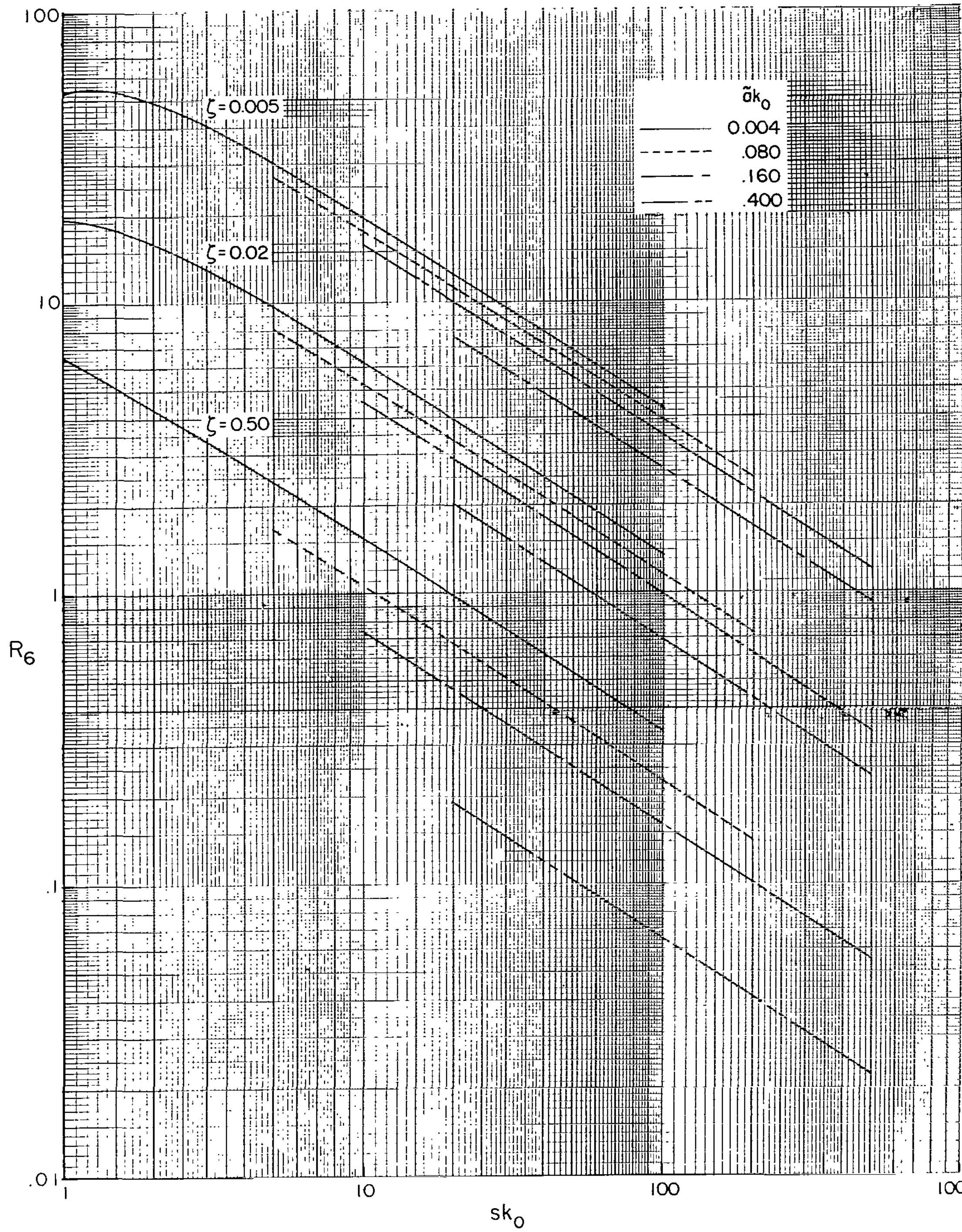


Figure 4.- Response integral R_6 as function of sk_0 for various values of ζ and $\tilde{a}k_0$.

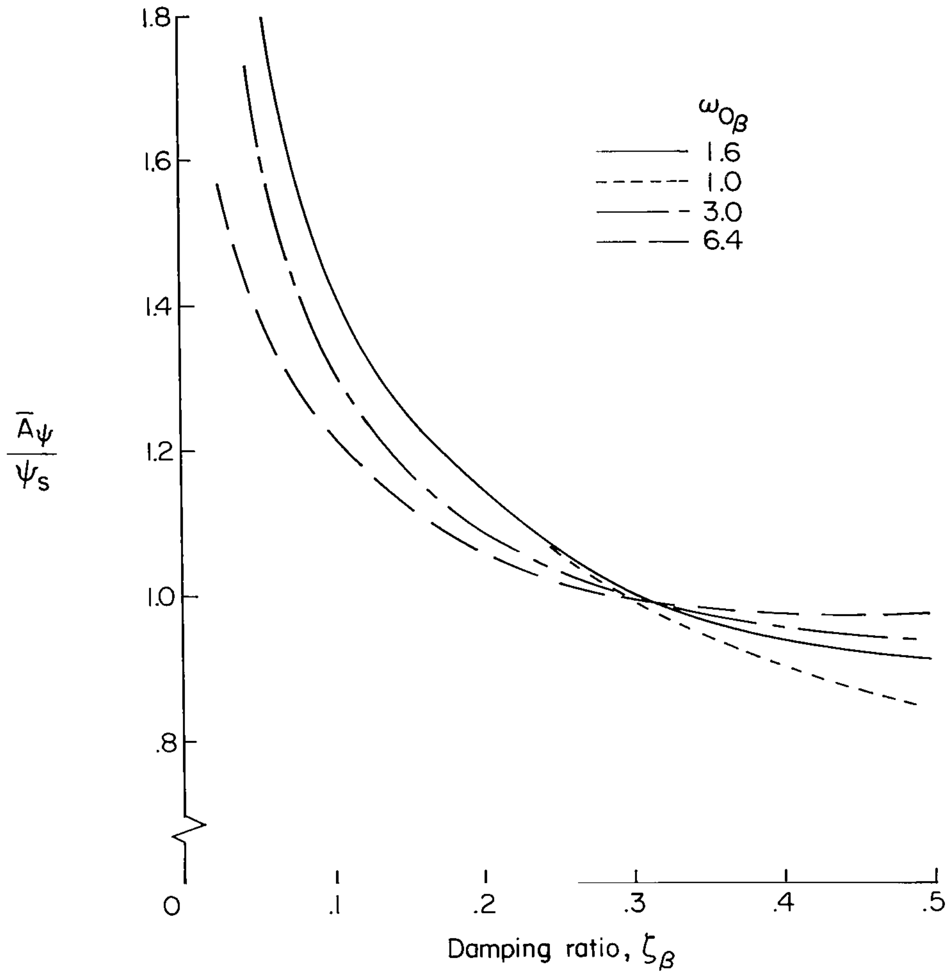


Figure 5.- Variation of $\overline{A}_\psi/\psi_s$ with ζ_β at various values of ω_{0_β}.

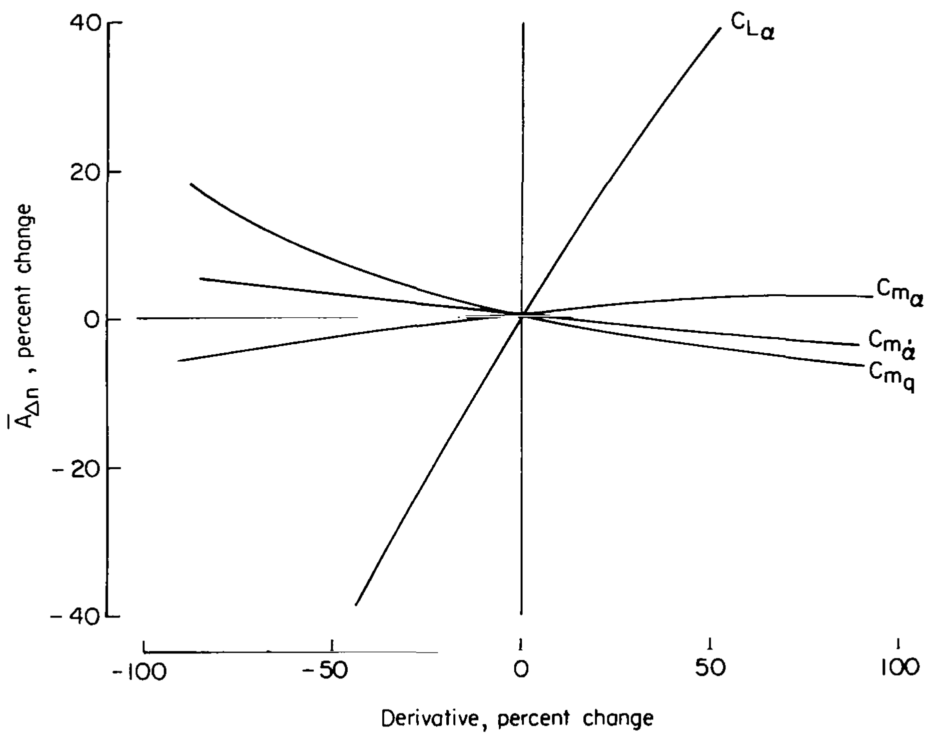
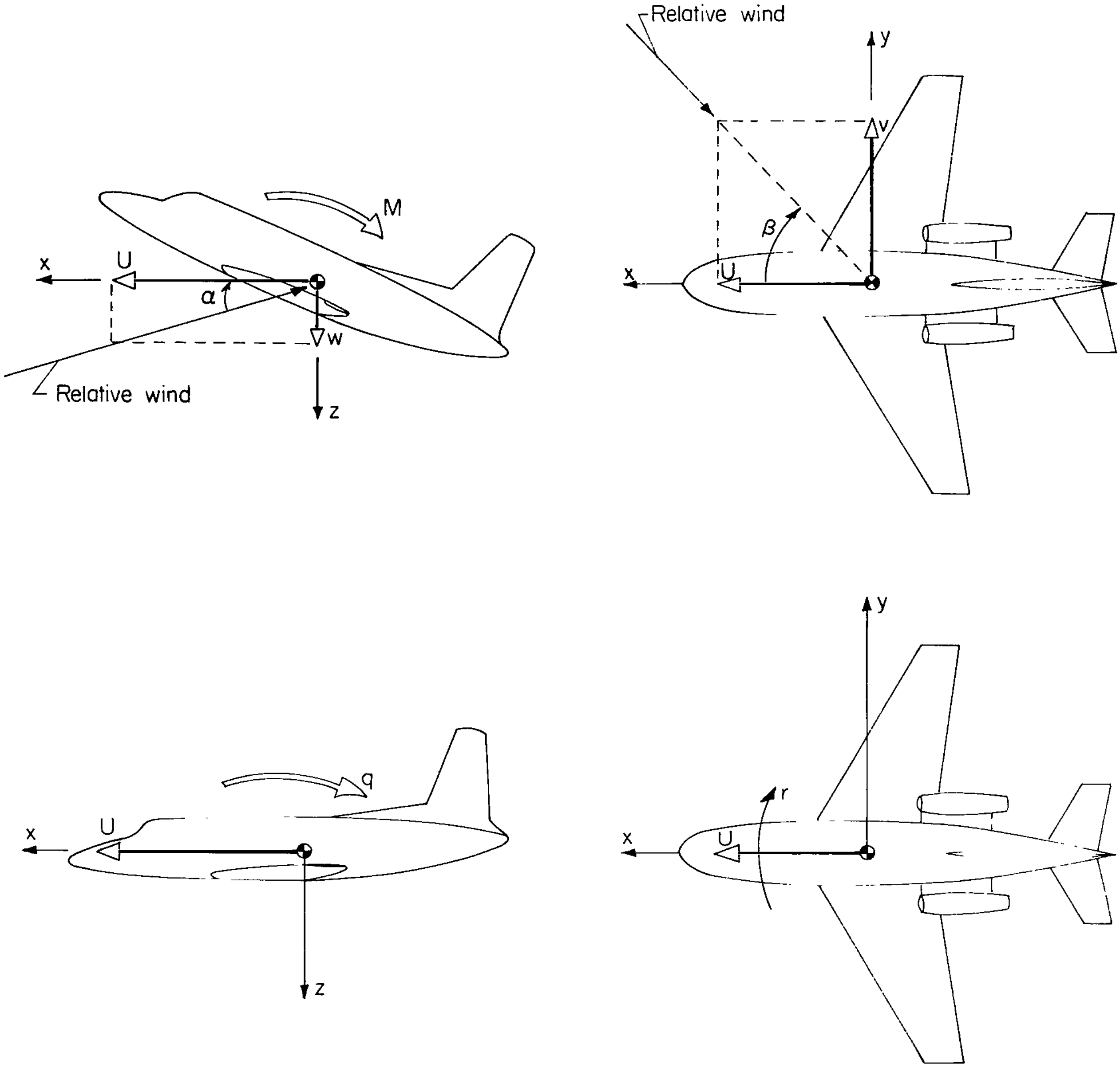


Figure 6.- Effect of stability derivatives on $\bar{A}_{\Delta n}$.



(a) Longitudinal axis.

(b) Lateral axis.

Figure 7.- Stability axes.

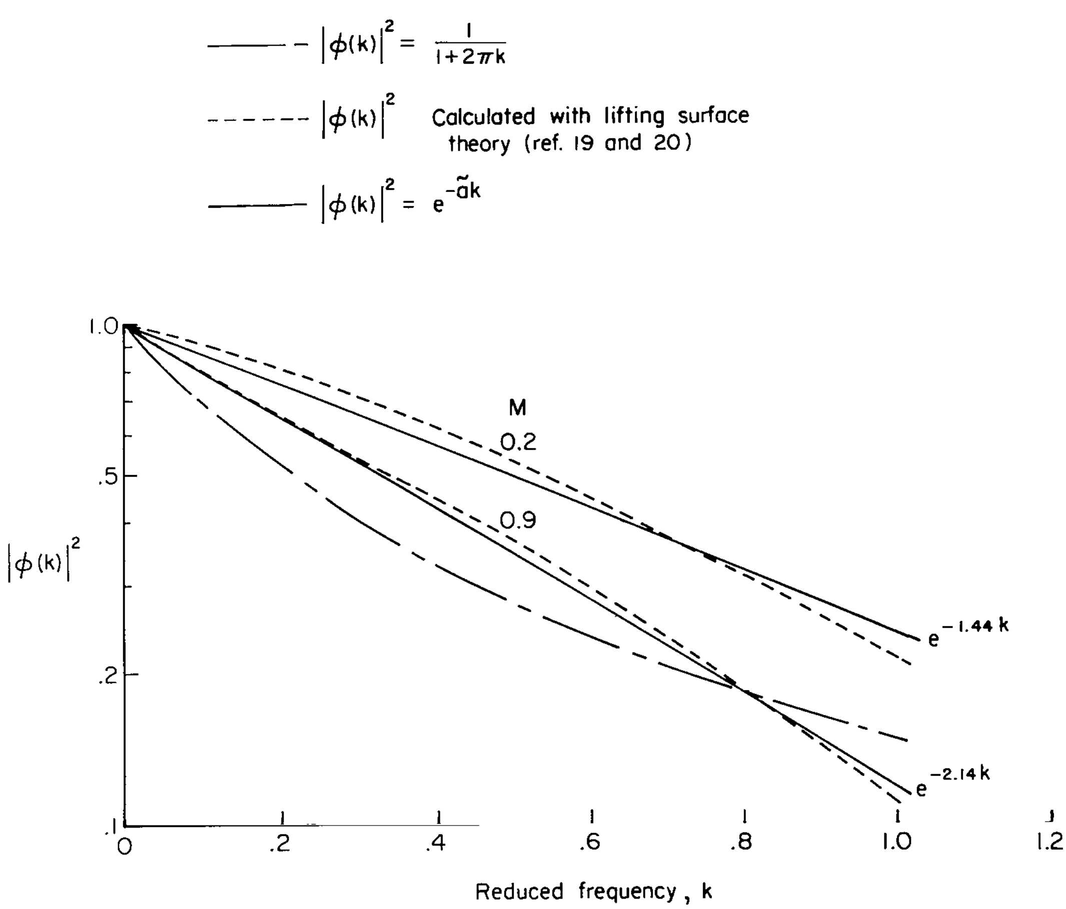


Figure 8.- Comparison of approximations for $|\phi(k)|^2$.

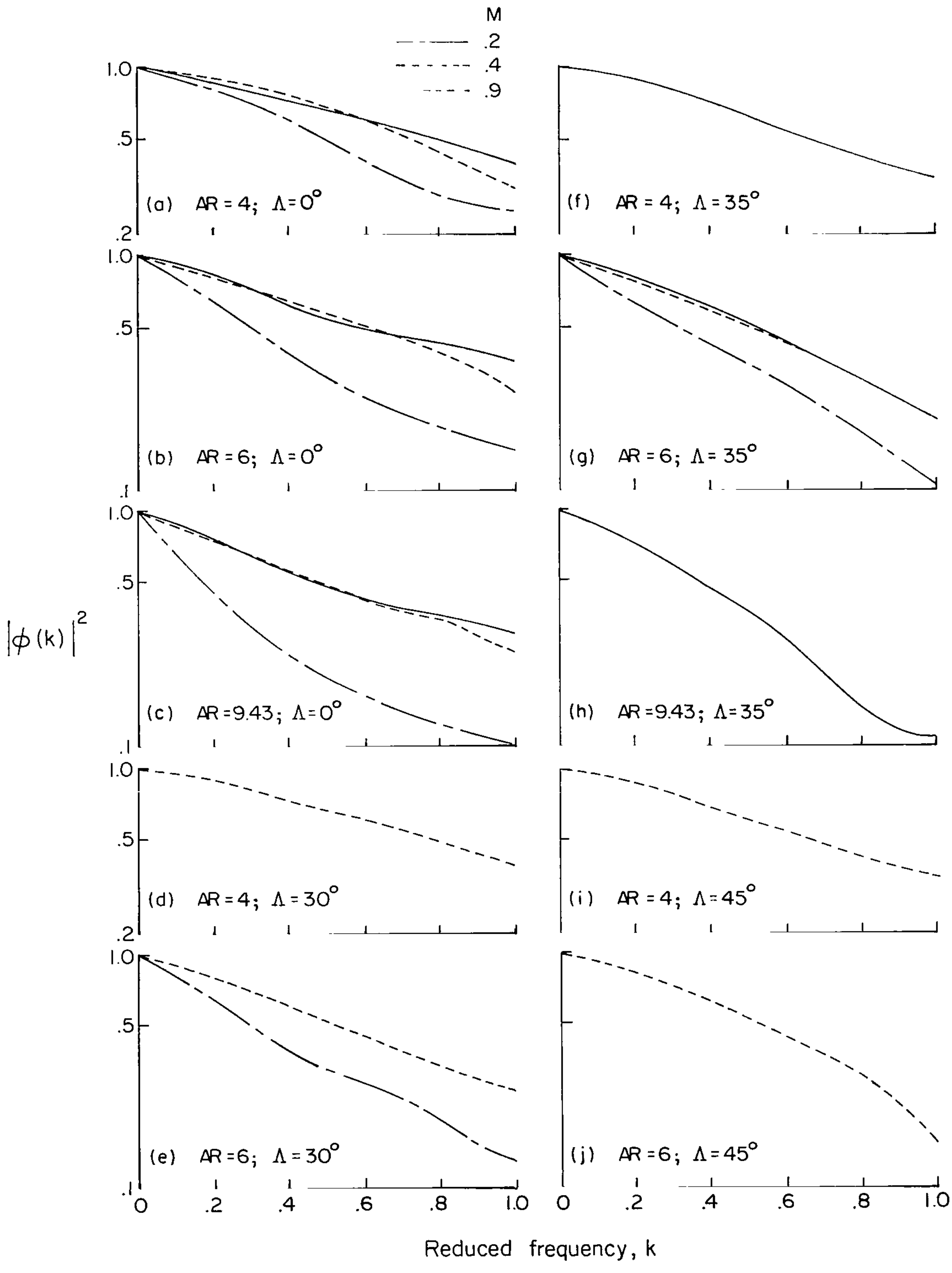


Figure 9.- Unsteady lift functions for various planforms and Mach numbers. (Some data were obtained from ref. 20.)

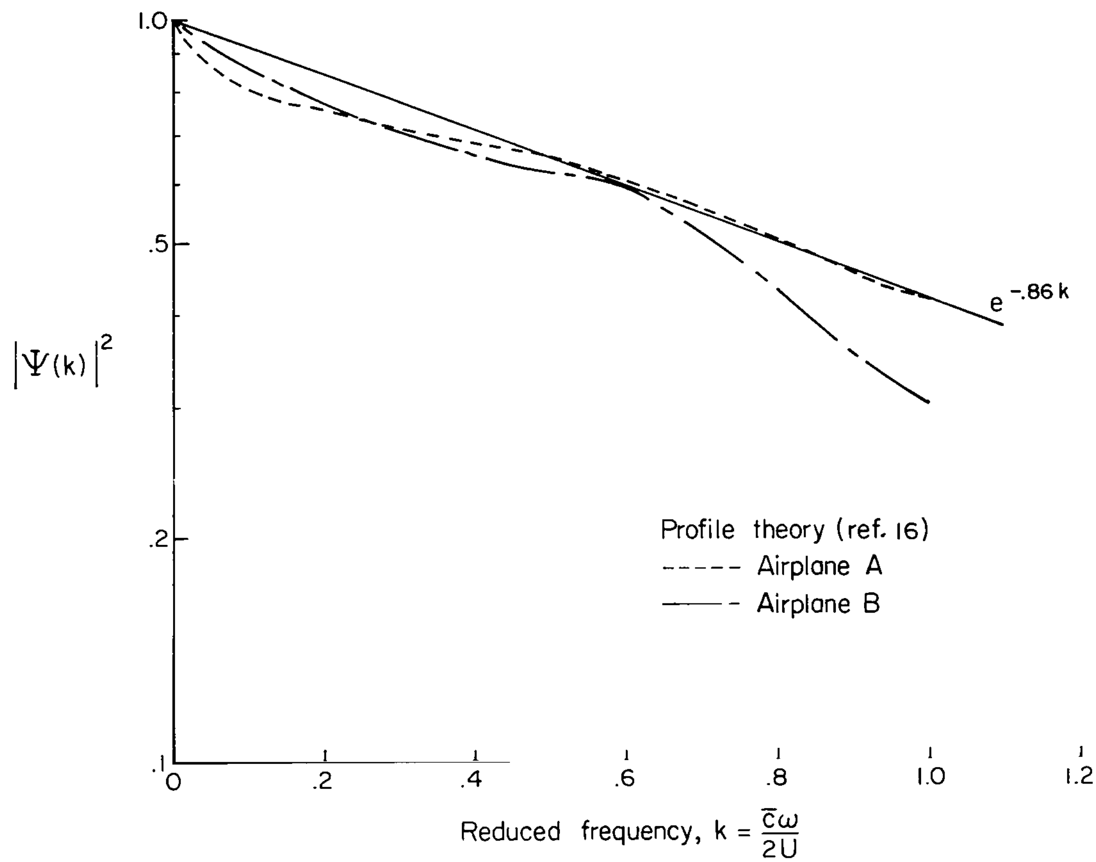


Figure 10.- Representative unsteady side-force functions.

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